Fracturing across the Multi-scales of Diverse Materials

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The root of the word ”fracture” comes from the Latin verb “frango” – “I break”; following the sequence “frango, frangere , fragi, fractum” we have the words ‘frangible’, fragile” and, of course, “fracture”. Fracture processes in a wide range of materials play a much larger role in everyday life than is commonly appreciated. In the home, we encounter fracture, not just when we drop glasses or crockery onto hard floors, or when a misdirected cricket ball smashes a window, but every time we tear off a sheet of aluminium foil or “cling-film”; every time we chop meat, peel vegetables, crack nuts, open sealed envelopes or packages, have our hair cut, shave, or trim our nails. Fractures in the wider world outside can be life-enhancing or events that cause great consternation. Aesthetic examples of the former are associated with the sculptures or wood-carvings that adorn our public places, civic and ecclesiastical buildings. Take, for example, Michelangelo’s “David” (Florence 1501-04). The production of this masterpiece involved fractures at two scales. First, a large slab of (Carrara) marble had to be quarried; second, Michelangelo had to produce a myriad of small brittle fractures in the marble slab to produce the finished form.

The quarrying of stone or the felling of trees is an essential pre-cursor to the craftsman producing final shapes by means of many small fractures. The craftsman also has to have confidence in his starting material: the stone must not contain internal cracks or pores that cause ugly cavities to be revealed, or lumps to break off, during the sculpting process; the wood must not contain knot-holes or the like that disfigure the final carving. A jeweler must take especial care when cleaving a raw diamond *en route* to producing a gemstone. In more utilitarian applications, the sawing of timber represents another common set of fractures and, in more general mechanical engineering applications, the machining of screw-threads into nuts, bolts, and screws represents millions upon million of ductile fractures. One of the most dramatic examples concerns the fracture of eggshells. We all know that “You can’t make an omelet without breaking eggs”, and, whether as omelets, boiled, fried, scrambled or in a cake mix; something of the order of a billion eggs are fractured and eaten every day. But the real purpose of an eggshell is to provide nurture and protection for bird chicks and for the young of some other species, such as turtles, alligators and the duck-billed platypus. All have to begin independent life by fracturing the eggshell from within. Mammals and human beings begin independent life by the fracture (“severing”) of the umbilical cord. Fracture is ubiquitous and essential to life. Yet the downside of fracture is that a number of the structures and machines upon which we rely so heavily for our well-being and comfortable life-styles have, from time to time, fractured unexpectedly, causing great upheaval and, often, loss of life.

Among the more widely known historical fracturing events have been wire cable breakages in mine shafts; failure of ‘Liberty’ ships and ‘Comet’ airplanes; collapse of the Johnstown, PA, dam and other assorted highway and rail bridges; and, most recently, the 9-11 skyscraper impact failures and the tsunami-induced catastrophe at Fukushima. And these civilian examples are overwhelmed in number by those caused as a result of military actions. Individual curiosities about why things should break have come a long way in the scientific and engineering communities. In the same way that the craftsman has to be aware of possible defects in his slab of marble or piece of wood, the engineer has to be aware that his fabricated structure may contain defects. A marked advance over the last 50-60 years has been the development of quantitative means of treating these defects: the engineering science of Fracture Mechanics. In turn, the properties of the materials depend on their microstructures and deformation mechanisms at the microscopic scale, and, for any fracture to occur, there must be mechanisms by which atomic bonds break. Events occur at different scales.

An early example of the probabilistic nature of fracture is found in Da Vinci’s work on the strength of iron wire (circa 1487) in which he found that the strength depended upon the length of the wire. These results have been interpreted as being due to the increased probability of finding a critical defect the longer the wire. The method of making wire in the Middle Ages was particularly prone to defects as were methods for obtaining the basic materials. Da Vinci’s results may be regarded as the forerunner of probabilistic fracture mechanics. Materials, test methods and analytical approaches have greatly improved. However, the so called “size effect” in fatigue of nickel-base superalloys used in jet engine disks is a well-known phenomenon linking back to Da Vinci’s experiments. The reduction in the fatigue life is explained as being due to larger specimens having a higher probability of containing a defect which causes early crack nucleation.

For the current themed issue, we draw inspiration from one of the early Fellows of the Royal Society: Robert Hooke. Hooke had many interests, but in the field of structural engineering, these ranged from the very large scale to the microscopic scale [1]. In his collaboration with Sir Christopher Wren on the re-building of St. Paul’s cathedral, he made a major contribution at the macroscopic scale, by suggesting that the dome should take the form of a “cubico-parabolical conoid”. In Hooke’s words (translated from his cipher) “ut pendet continuum flexile sic stabit grund Rigidum” [2] (see also Lisa Jardine’s biography of Wren “On a Grander Scale”). Hooke was also aware of the fracture of brittle building materials and he made painstaking observations of the fracture surfaces of the oolitic limestone to be used for St. Paul’s, using a microscope, but taking sequential focus positions, so that he was able to produce ‘three-dimensional” representations in his 17th Century Micrographia publication as shown on the RS theme issue cover page. From “The Grander Scale” to the microscopic scale!

Hull has researched Hooke’s work on this topic and obtained interesting information at the time on use of the material for building construction in Cambridge and on wider interest in fracturing [3]. Only one century later, Reaumur was to comment from observation of fracture surfaces that excellent steels were distinguished from mediocre steels according to the fineness of their grains [4]. This was some 130 years before the Bessemer steelmaking process came into being: Reaumur had his own process for making steel (for which he was awarded a prize by the French Government). Hull mentions even later 19th century interest of Sorby in geological materials [5]. Sorby became so absorbed in founding the microscopic examination of sectioned metal microstructures that he made the following comment [6]: *“Compared with what can be learned from good sections, the study of mere fractures teaches very little respecting the ultimate structure because a fracture occurs along a line of weakness between the constituent grains, whilst a section shows the true relation and ultimate constitution of the constituent crystals”*. In one sense, the difference is determined by what defect-free material condition should control the fracturing property as compared with what *local defect condition* did control the observed condition of failure. And for an example of modern fractographic analysis, one can quote James [7]: “*The fracture surface contains a complete record of the events experienced by the component during fracture and the skill in fractography comes in understanding and interpreting those features in a clear description of causes and mechanisms involved in the cracking process.*” Lynch and Moutsos [8] have given an important review of fractographic images dating back to observations made in the 18th century by Reaumur.

Armstrong has previously established modern connection with Reaumur’s observations relating to the influence of polycrystal grain size in initially defect-free material on determining the brittle fracturing behavior of steels [9]. More recently, Knott produced a quantifying description of steel quality in his 2008 Hatfield Memorial Lecture [10]. Otherwise, the internal structural relationship of the mechanical properties of (mostly ionic) mineral *versus* those of metallic crystals had continued to be described in a common manner, for example as contained in the seminal book produced in the first third of the 20th century by Schmid and Boas [11]. Each subject then became so large and diversified that information on the two types of material behaviour has proceeded to develop essentially on its own. As mentioned above, one purpose of the present theme issue is to contribute to ‘bridging the gap’.

Another purpose of the present theme issue is to deal with the dimensional scale at which fracture-controlling events occur and are observable with modern atomic resolution capabilities. A recent review of observations made of hardness indentations put into various crystal, polycrystal, polyphase and amorphous material surfaces, often with associated cracking, has covered a smallest dimensional scale from 0.06-0.07 nm to ~1.0 mm [12]. The range in dimensional scale coincides with that observed for the internal organization of (crystal) grains in material structures, although cast heavy steel plate material is known to exhibit dendritic solidification heterogeneity on a scale of cms or larger as apply for the same dimensions employed in reinforced concrete and other larger scale engineering structures. But even for the further scaling up of real macro-engineering structures, it should be remembered that the same understanding of fracturing behavior applies for the lower specified dimensional scale. A recent report [13] providing a thermal cycling explanation of cracking origin for the regolith covering the 54 km length of the chondritic asteroid "Ida" relied on the 'Paris Law' whose development was motivated by the need to predict the fatigue crack growth rate in airframes [14][1]. Even more recently, an account has been given of the widely-witnessed explosive fracturing of the half-megaton asteroid at 45-40 km over Chelyabinsk [15], and deduced to have occurred at a dynamic (air) pressure in the range of 0.7-1.0 MPa. For contrast at opposite material and dimensional scale, the mechanisms determining resistance to fracturing of bone materials have been described recently at length scales ranging from the nano- to meso-scale in the hierarchical structure [16].

The theme issue begins with the physics and mechanics of fracturing at the largest dimensional scale. Great progress was made by Inglis [17] and Griffith [18] in the beginning of the 20th century in taking account of the effect of pre-existent cracks on determining the fracture strength of steel and glass materials. Irwin and colleagues carried the work forward in the mid-20th century with the development of the subject of *Fracture Mechanics* [19]. Cherepanov [20], Rice [21], Atkinson and Eshelby [22] and colleagues have put *FM* on a firm mathematical foundation. The dynamics of crack growth given by Mott [23] have been extended by Rabinovitch [24] employing the condition of crack length being larger than determined by the Griffith criterion. Frank and Lawn [25] provided a Griffith-based analysis of cone-cracking at indentations in glass materials and which analysis was rapidly extended to other indentation crack geometries [12].

Plastic flow considerations enter both into the nucleation of cracks and the nature of crack extensions, in the latter situation, no matter how brittle the material may appear to be. Thus the crystal dislocation based plasticity theory co-invented just earlier in the 20th century by Taylor [26], Orowan [27] and Polanyi [28] has been carried forward with important relevance to fracture, for example, in the work of Orowan [29], Petch [30], Cottrell [31, 32], Crussard and colleagues [33, 34], Yokobori *et al*. [35], Nabarro [36], Kochendorfer *et al*. [37], Friedel *et al*. [38] and Hirsch *et al*. [39], among many other colleagues. Current developments include such computational 'code' calculations as mesoscale cracking simulations to elucidate material deformation and cracking behaviors [40]. A recent example of simulated atomic-scale plasticity initiated at nano-scale holes in tantalum crystals is given by Tramontina, Ruestes, Tang and

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[1] Paris' major contribution was to recognize that, independent of geometry, ΔK provides a practical engineering forcing function for fatigue crack propagation. Some form of da/dN *vs* ΔK is now widely used for life prediction; however Paris' considerable difficulties in gaining publication resulted in an unusual publication outlet!

Bringa [41] for which a comparison is made of molecular dynamics results obtained using two different empirical lattice potentials.

The ubiquitous aspect of fracturing events and accompanying processes first-mentioned covers a broader range of interest than could be contained in the present theme issue, even for the range in dimensional scale and diversity of materials that have been reported on. The omissions are regretted. In the current project, overlapping contributions were sought in a number of subject areas beginning with the issue of *macro-scale fracturing* as put forward in the article by Cherepanov [42]. Related references [43]-[46] are added here to provide examples of complementary analyses extending from a latest report on fracking through mining technology to geophysical aspects of snow avalanching and cracking within glaciers.

Towards the 'opposite' *crystal/polycrystal/nanopolycrystal level* of size scale, coverage is provided by Antolovich [47], Armstrong [48], Hohenwarter and Pippan [49], Ovid'ko [50], and Pineau [51]. Emphasis is given by these authors to the importance of crystal/grain boundaries. Useful information on engineering of grain boundary structures is in the supplementary references [52], [53]. Lower limiting dimensional considerations are described for *atomic-scale fracturing* by Brenner and Shenderova in the case of diamond [54] and by Rouxel for glass materials [55]. The jeweler's cleavage of a diamond is analysed at the atomic scale.   Hird *et al.* report on the perceived construction of diamond lenses for historical eye-glasses [56] as compared with Lodes, Kachold and Rosiwal reporting on the fracturing of smaller crystals in diamond foil [57]. There is interesting local atomic order/amorphous material consideration in reference [58] and corresponding extension to *amorphous material aspects* of fracturing in references [59] and [60].

The *dynamics of fracturing* at highest crack speeds are theoretically modeled at the atomic scale by Behn and Marder [61] and experimentally described for projectile impacts on glass materials by Chaudhri [62]; rather more sophisticated than a cricket ball smashing a glass window. Related theoretical analyses in the former case are in references [63], [64] and, for glass material, to essentially static hardness testing by Rouxel [55] and applied to hierarchical structural aspects of tooth enamel by Yilmaz *et al*. [65]. Other benefits of structural considerations dealing with experimental fractographic analyses, building onto Hull's attribution to Hooke [3], are provided in references [66], [67], relating also to an exposition of *ductile versus brittle fracturing* considerations in the articles by Tekoglu *et al*. [68]; Armstrong [48]; Knott [69]; and Matic, Geltmacher and Rath [70].

Early promotion of fracture mechanics aspects of the topic [19], [71], including microstructural aspects [72], were carried forward in many research articles, for example, in reference [73]. Quite a few years earlier, Orowan [29] had remarked in pioneering work that fracturing is a mechanism-sensitive process, hence such added descriptions as 'fatigue fracturing', 'creep fracturing', etc. Such consideration applies to any number of different type observations made in the already referenced articles [42], [49], [50], [53], [59] and relates to other *variable stress aspects* of controlled fracture testing as exemplified in the articles by Atkinson, Coman and Aldazabal [74]; Matic *et al*. [70]; and Mughrabi [75].

And lastly, we co-editors mention our pleasure to have organized the current compilation of reports on fracturing, including those sub-topics, contained in the current theme issue.

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